Definitions for Probability

Probability

Probability is the likelihood of the occurance of an event. The probability of event A is written P(A). Probabilities are always numbers between 0 and 1, inclusive.

The four basic rules of probability:

1) For any event A, $0 \leq P(A) \leq 1$.
2) P(impossible event) = 0.
   Also written P(empty set) = 0 or $P(\emptyset) = 0$.
3) P(sure event) = 1.
   Also written P(S) = 1, where S is the sample set.
4) P(not A) = 1 - P(A).
   Also written P(complement of A) = 1 - P(A) or $P(A^C) = 1 - P(A)$ or $P(\bar{A}) = 1 - P(A)$.

Experiment

In the study of probability, the name given to any controlled and repeatable process.

Event

A set of possible outcomes resulting from a particular experiment.

Outcome

A single specific possible result of an experiment.

Experiment	Outcomes
Tossing a coin	Heads, Tails
Rolling a six sided die	1,2,3,4,5,6

Probability : Independent Events

Independent Events

Independent Events are not affected by previous Events.

A coin does not "know" it came up heads before ...

... each toss of a coin is a perfect isolated event.

When rolling a pair of dice, one die does not affect the outcome of the other die ...

... each die is an isolated event.

$$\textbf{Probability of an event happening} = \frac{\textbf{Number of ways it can happen}}{\textbf{Total number of outcomes}}$$

Probability of getting a "Head" when tossing a coin?

$$P(\text{Head}) = \frac{\text{"Head"}}{\text{"Head and Tail"}} = \frac{1}{2}$$

Probability of rolling a "4" on a die?

$$P(4) = \frac{\text{"4"}}{\text{"1","2","3","4","5","6"}} = \frac{1}{6}$$

Probability : Independent Events

Two or More Events

You can calculate the probability of two or more Events by multipling the individual probabilities.

So, for Independent Events:

P(A and B) = P(A) x P(B)

Example: Probability of 3 Heads in a Row

For each toss of a coin a "Head" has a probability of 0.5 :

0.5

0.5 x 0.5 = 0.25

0.5 x 0.5 x 0.5 = 0.125

So the Probability of getting three Heads in a Row is 0.125.

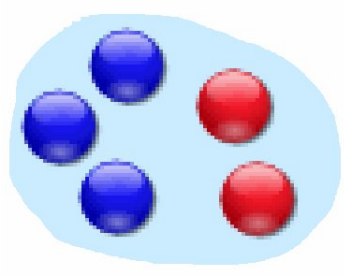
Conditional Probability : Dependent Events

Dependent Events

Dependent Events are affected by previous events.

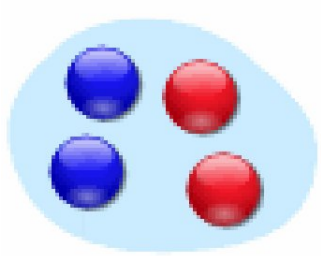
Example: Marbles in a Bag

There are 3 blue and 2 red marbles in a bag.
What is the probability of drawing a blue marble on the first and second draw?



$$P(\text{Blue 1st Draw}) = \frac{3}{5}$$

after the first draw you have changed the chances for the next draw



$$P(\text{Blue 2nd Draw}) = \frac{2}{4} = \frac{1}{2}$$

The probability of

$$P(\text{Blue 1st Draw and Blue 2nd Draw}) = P(\text{Blue 1st Draw}) \times P(\text{Blue 2nd Draw})$$

$$P(\text{Blue 1st Draw and Blue 2nd Draw}) = \frac{3}{5} \times \frac{1}{2}$$

$$P(\text{Blue 1st Draw and Blue 2nd Draw}) = \frac{3}{10}$$

Replacement

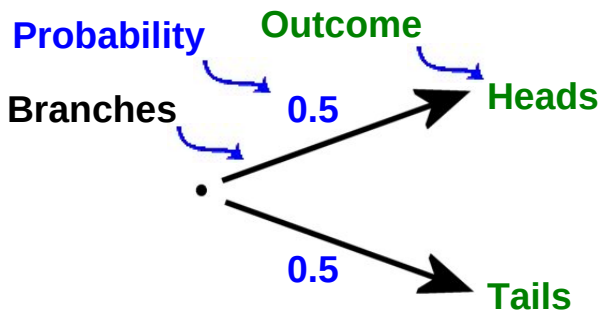
Note: if you had replaced the marbles in the bag each time, then the chances would not have changed and the events would be independent:

- With Replacement: the Events are Independent (the chances don't change)
- Without Replacement: the Events are Dependent (the chances change)

Probability Tree Diagrams

Calculating probability is confusing at times, especially for multiple events.
Tree diagrams give you a visual and more simple way to solve complex probability problems.
The diagrams are composed of three items: Branches, Probabilites, and Outcomes.

Example: Probability of tossing a coin.

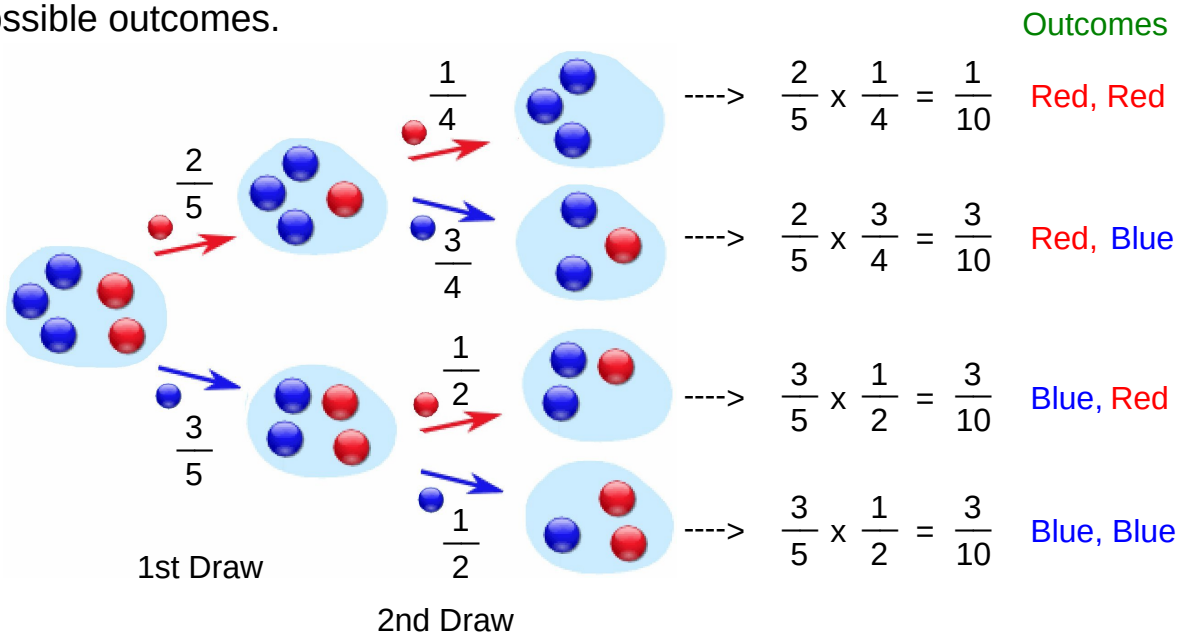


There are two Branches (Heads and Tails)

- The probability for each Branch is written on the Branch (0.5)
- The Outcome is written at the end of the Branch.

Now let us graph the previous example of the 3 blue and 2 red marbles in a bag.
What is the probability of drawing two red marbles from the bag with no replacements?

Graph out all possible outcomes.



You may find the probability of any outcome by multiplying the probabilities along any path.